HOLIDAY HOME WORK FOR CLASS XII

RELATIONS AND FUNCTIONS

1. Let m be a positive integer. For integers a,b we say that they are congruent modulo m
iff a-b is divisible by m. We write this as $a \equiv b (\mathrm{mod}\, m)$.
Let R be the relation on the set Z of integers defined by aRb iff $a \equiv b (\mathrm{mod}\, m)$. Show that
R is an equivalence relation.

2. If $f(x) = (a - x^n)^{\frac{1}{n}}$,prove that f(f(x))=x.

3. Let $f: X \rightarrow Y$ be an invertible function, Show that f has unique inverse.

4. If R_1 and R_2 are equivalence relation in a set A,Show that $R_1 \cap R_2$ is an equivalence relation.

5. Prove that the greatest integer function $f: R \rightarrow R$ given by f(x)=[x] is neither one-one nor onto,
Where [x] denotes the greatest integer less than or equal to x.

INVERSE TRIGONOMETRIC FUNCTIONS

1. Find the value of $\tan\left(\frac{1}{2}\left(\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right)\right), |x| < 1,\ y > 0,\ xy < 1$

2. Write the simplest form of, $\cos^{-1}\left(\frac{1-x}{1+x}\right)$

3. Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

4. Solve for x: $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

5. Prove that $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$

MATRICES

1. If A= $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB and use this to solve the following

System of equations. $\begin{matrix} x - y + z = 4 \\ x - 2y - 2z = 9 \\ 2x + y + 3z = 1 \end{matrix}$

2. If A= $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then prove by mathematical induction that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for every positive integer n.

3. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

4. Find k so that $A^2 = kA - 2I$. Where $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$.

5. Find inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary operation.

DETERMINANTS

1. Using properties of determinants, Show that $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

2. If none of a,b and c is zero, using properties of determinant prove that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (bc+ca+ab)^3$$

3. Using matrix method to solve the following system of equation

$$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4, \frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0, \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2, \qquad x,y,z \neq 0$$

4 . Using properties of determinant prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

5. Using properties of determinant prove the $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (ab+bc+ca)(a-b)(b-c)(c-a)$